Review – Polynomial Division and Graphing Rational Functions

Percent:_____

Name: Key
Date:
Period:

Graph each of the following. Find all necessary features, skip features that are not needed to graph.

1. $f(x) = \frac{2x}{x-1}$

x- intercept(s):

$2x = 0$

$(0,0)$

y-intercept:

$f(0) = \frac{0}{0-1} = 0$

$(0,0)$

Vert. Asymp/Holes:

$x - 1 = 0$

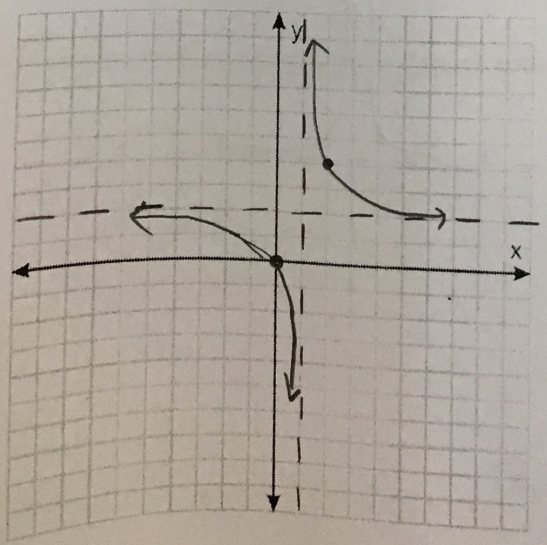
$x = 1$

Horz Asymp:

$\frac{2}{1} = y = 2$

Extra Points (if necessary):

$f(2) = \frac{2(2)}{2-1} = \frac{4}{1} = (2,4)$



2. $g(x) = \frac{4x}{x^2-36} = \frac{4x}{(x+6)(x-6)}$

x- intercept(s):

$4x = 0$

$x = 0$

$(0,0)$

y-intercept:

$f(0) = \frac{0}{0-36} = (0,0)$

Vert. Asymp/Holes:

$x = -6$

$x = 6$

Horz Asymp:

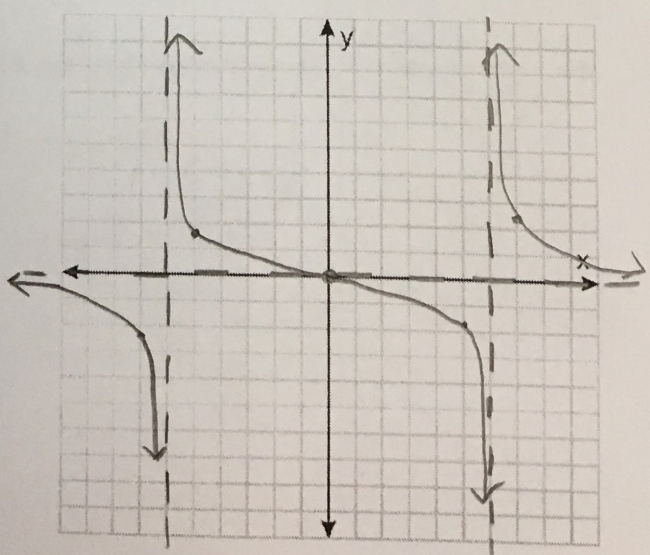
Top lower degree $\rightarrow y = 0$

Extra Points (if necessary):

$f(5) = \frac{4(5)}{25-36} = \frac{20}{-11} = (5, -\frac{20}{11})$

$f(-5) = \frac{4(-5)}{25-36} = \frac{-20}{-11} = (5, \frac{20}{11})$

$f(7) = \frac{4(7)}{49-36} = \frac{28}{13} = (7, \frac{28}{13})$



3. $f(x) = \frac{x^2+2x-3}{x-2} = \frac{(x+3)(x-1)}{x-2}$

x- intercept(s):

$(x+3)(x-1)=0$

$x=-3 \quad x=1$

$(-3,0) \quad (1,0)$

y-intercept:

$f(0) = \frac{0+0-3}{0-2} = \frac{3}{2}$

$(0, \frac{3}{2})$

Vert. Asymp/Holes:

$x=2$

Horz Asymp: slant $y=x$

$$x-2 \overline{)\, x^2+2x-3} \quad (x+4)$$

$$-x^2+2x$$

$$+4x$$

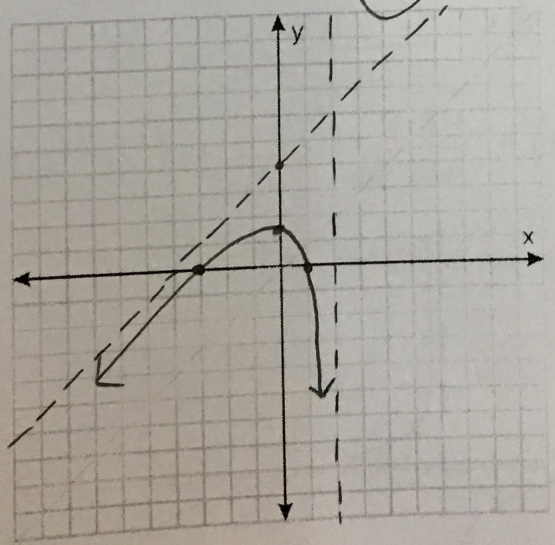
Extra Points (if necessary):

$f(3) = \frac{(3)^2+2(3)-3}{3-2}$

$= \frac{9+6-3}{1}$

$= (3,12)$

(3,12)



4. $g(x) = \frac{1}{(x+3)^2} - 4$

x- intercept(s):

y-intercept:

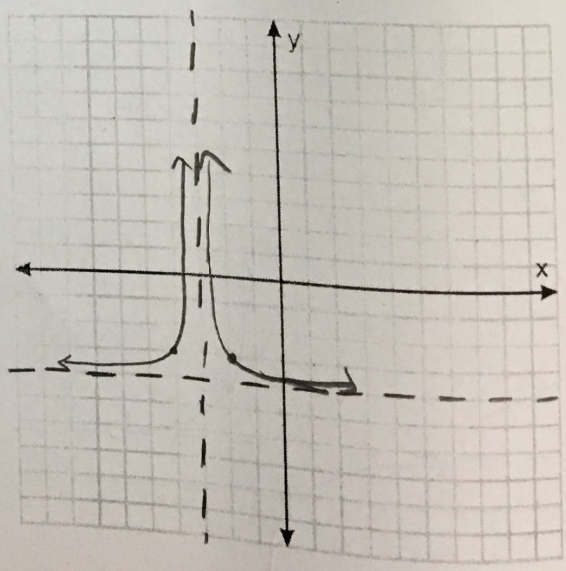
Vert. Asymp/Holes:

$x=-3$

Horz Asymp:

$y=-4$

Extra Points (if necessary):



State the domain of each rational function.

5. $f(x) = \frac{x+8}{x^2-64} = \frac{(x+8)}{(x+8)(x-8)}$

$\{x \mid x \neq -8, 8\}$

$(-\infty, -8) \cup (-8, 8) \cup (8, \infty)$

6. $f(x) = \frac{x+8}{x^2+64}$

$(-\infty, \infty)$

$\{x \mid x \in \mathbb{R}\}$

Use the graph to complete each statement.

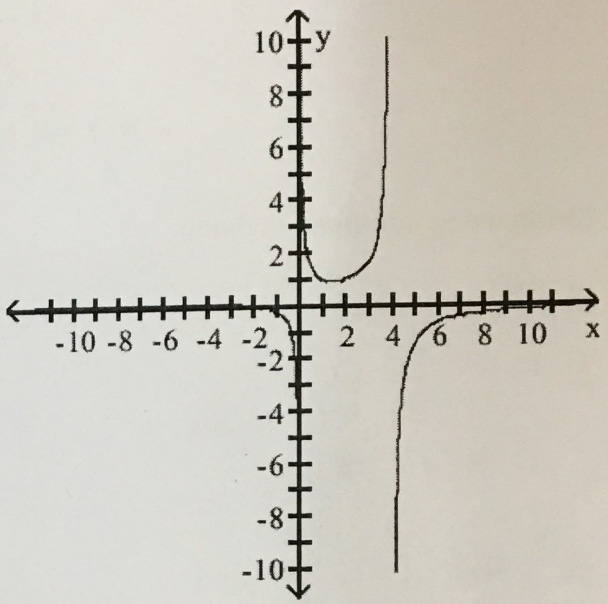
Change to 4^- and 4^+

7. $\lim_{x \to +4^-} f(x) = \infty$

8. $\lim_{x \to +4^+} f(x) = -\infty$

9. $\lim_{x \to -\infty} f(x) = 0$

10. $\lim_{x \to \infty} f(x) = 0$



Find the vertical asymptotes and/or holes of each of the following:

11. $f(x) = \frac{x^2-9}{x+3} = \frac{(x+3)(x-3)}{x+3}$

Hole at $x = -3$

12. $f(x) = \frac{x+6}{x(x^2-2x-24)} = \frac{x+6}{x(x+4)(x-6)}$

V.A. $x = 0$

$x = -4$

$x = 6$

Find the horizontal asymptote/slant asymptote of each of the following:

13. $f(x) = \frac{-2x+1}{3x+5}$

$y = -\frac{2}{3}$

14. $f(x) = \frac{x^2+4}{x}$

$y = x$

$$x \overline{)\, x^2 + 4} \quad (x)$$
$$-x^2$$
$$+4$$

Divide using long division.

15. $\frac{2x^5-8x^4+2x^3+x^2}{2x^3+1}$

$$2x^3+1 \overline{)\,2x^5-8x^4+2x^3+x^2+0x+0}$$

$$x^2-4x+1$$

$$-2x^3 \quad \mp x^2$$

$$-8x^4+2x^3+0x^2+0x+0$$

$$+8x^4 \quad +4x$$

$$2x^3+0x^2+4x+0$$

$$-2x^3 \quad \mp 1$$

$$0x^2+4x-1$$

$$= x^2-4x+1+\frac{4x-1}{2x^3+1}$$

16. $\frac{6x^3+13x^2-11x-15}{3x^2-x-3}$

$$3x^2-x-3 \overline{)\,6x^3+13x^2-11x-15}$$

$$2x+5$$

$$-6x^3+2x^2+6x$$

$$15x^2-5x-15$$

$$-15x^2+5x+15$$

$$0$$

Divide using synthetic division.

17. $\frac{x^4-256}{x-4}$

4	1	0	0	0	−256
		4	16	64	256
	1	4	16	64	0

$$= x^3+4x^2+16x+64$$

18. $\frac{x^2-6x-6x^3+x^4}{6+x}$

−6	1	−6	1	−6	0
		−6	72	−438	2664
	1	−12	73	−444	2664

$$= x^3-12x^2+73x-444+\frac{2664}{6+x}$$

Find all remaining ZEROS of the function, using the zero given.

19. Find all zeros of the equation $2x^3 - 5x^2 + x + 2 = 0$ given that 2 is a zero.

2	2	−5	1	2
		4	−2	−2
	2	−1	−1	0

$$2x^2-x-1=0$$

$$(2x+1)(2x-2)$$

$$(2x+1)(x-1)$$

$$x=-\tfrac{1}{2} \quad x=1 \quad x=2$$

Use synthetic division and the remainder theorem to find the indicated function value.

20. $f(x) = 2x^4 - 5x^3 - x^2 + 3x + 2; f\left(-\frac{1}{2}\right)$

$-\frac{1}{2}$	2	−5	−1	3	2
		−1	3	−1	−1
	2	−6	2	2	1

$$f\left(-\tfrac{1}{2}\right) = 1$$