Review – Exponential and Logarithmic Equations & Modeling

Score: ________ of ________

Percent: ____________

Name: Key
Date:
Period:

Solve the following exponential equation by expressing each side as a power of the same base and then equating exponents.

1. $16^{x+8} = 256^{x-8}$

$16^{x+8} = (16^2)^{x-8}$

$x+8 = 2x-16$

$24 = x$

2. $3^{6-x} = \frac{1}{9}$

$3^{6-x} = 3^{-2}$

$6-x = -2$

$-x = -8$

$x = 8$

Solve the given exponential equation. Express the solution set in terms of natural logarithms or common logarithms. Then use a calculator to obtain a decimal approximation to three decimal places for the solution.

3. $7e^x = 11$

$\frac{7e^x}{7} = \frac{11}{7}$

$e^x = \frac{11}{7}$

$x = \ln\frac{11}{7}$

$x \approx .452$

4. $e^{1-4x} = 1081$

$\ln e^{1-4x} = \ln 1081$

$1-4x = \ln 1081$

$-4x = \ln 1081 - 1$

$x = \frac{\ln 1081 - 1}{-4}$

$x \approx -1.496$

5. $5^{2x+5} = 3^{x-7}$

$\ln 5^{2x+5} = \ln 3^{x-7}$

$(2x+5)\ln 5 = (x-7)\ln 3$

$2x\ln 5 + 5\ln 5 = x\ln 3 - 7\ln 3$

$2x\ln 5 - x\ln 3 = -5\ln 5 - 7\ln 3$

$x(2\ln 5 - \ln 3) = -5\ln 5 - 7\ln 3$

$x = \frac{-5\ln 5 - 7\ln 3}{2\ln 5 - \ln 3}$

$x \approx -7.422$

6. $e^{2x} - 6e^x + 5 = 0$

$(e^x - 5)(e^x - 1) = 0$

$e^x = 5 \quad e^x = 1$

$x = \ln 5 \quad x = \ln 1$

$x \approx 1.609 \quad x = 0$

Solve the logarithmic equation. Be sure to reject any value of x that is not in the domain of the original logarithmic expression.

7. $8\ln(9x) = 16$

$\ln(9x) = 2$

$\log_e 9x = 2$

$e^2 = 9x$

$x = \frac{e^2}{9}$

$x \approx .821$

8. $\log_4(6x+4) = 6$

$4^6 = 6x + 4$

$4096 = 6x + 4$

$4092 = 6x$

$682 = x$

9. $\log_4(x+7) + \log_4(x+4) = 1$

$\log_4(x+7)(x+4) = 1$

$\log_4(x^2+11x+28) = 1$

$4^1 = x^2+11x+28$

$x^2+11x+24=0$

$(x+8)(x+3)$

$x=-8$ $x=-3$

makes $(x+4)<0$ cannot take log of − #.

10. $\ln(x-2) - \ln(x+5) = \ln(x-1) - \ln(x+10)$

$\ln\frac{x-2}{x+5} = \ln\frac{x-1}{x+10}$

$\cancel{(x+5)}\frac{x-2}{\cancel{x+5}} = \frac{x-1}{x+10}\cdot(x+5)$

$(x+10)\,x-2 = \frac{x^2+4x-5}{\cancel{x+10}}\cancel{(x+10)}$

$x^2+8x-20 = x^2+4x-5$

$-x^2-4x+20 \quad -x^2-4x+20$

$4x = 15$

$x = \frac{15}{4}$

Find the time, in years, necessary to reach the amount given based on a savings account with n compounding periods and the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$. Round answers to one decimal place.

11. How long will it take an initial investment of \$7250 to accumulate to \$15000 with a 6.5% interest rate compounded monthly?

$15000 = 7250\left(1+\frac{.065}{12}\right)^{12t}$

$\frac{15000}{7250} = \left(1+\frac{.065}{12}\right)^{12t}$

$$\frac{\dfrac{\ln\frac{15000}{7250}}{\ln\left(1+\frac{.065}{12}\right)}}{12} = t$$

$t \approx 11.2$ years.

12. in 2000 the population of a country was approximately 6.02 million and by 2015 it is projected to grow to 7 million. Use the exponential growth model $A = A_0 e^{kt}$, in which t is the number of years after 2000 and A_0 is in millions, to find an exponential growth function that models the data.

$7 = 6.02e^{k(15)}$ $\quad k \approx .0101$

$\frac{7}{6.02} = e^{15k}$

$\frac{\ln \frac{7}{6.02}}{15} = k$ $\quad A = 6.02e^{.0101t}$

b. By which year will the population be 10 million? Round to a whole number.

$10 = 6.02e^{.0101t}$ $\quad t \approx 50$ years

$\frac{10}{6.02} = e^{.0101t}$ $\quad$ Approx. in 2050.

$\frac{\ln \frac{10}{6.02}}{.0101} = t$

13. The half-life of a certain substance is 17years. How long will it take for a sample of this substance to decay to 73% of its original amount? Round to one decimal place as needed.

$\frac{1}{2}A_0 = A_0 e^{kt}$

$\frac{1}{2} = e^{k(17)}$

$\frac{\ln \frac{1}{2}}{17} = \frac{\cancel{17}k}{\cancel{17}}$

$\frac{\ln \frac{1}{2}}{17} = k$

$-.0408 = k$

$\boxed{A = A_0 e^{-.0408t}}$

$.73A_0 = A_0 e^{-.0408t}$

$.73 = e^{-.0408t}$

$\frac{\ln(.73)}{-.0408} = t$

$\boxed{t \approx 7.7 \text{ years}}$

14. The logistic growth function $f(t) = \frac{102{,}000}{1+5900e^{-t}}$ describes the number of people, f(t), who have become ill with influenza t weeks after its initial outbreak in a particular community.

a. How many people became ill when the epidemic began?

$f(0) = \frac{102{,}000}{1 + 5900e^{(0)}}$

Approx. 18 people.

b. How many people were ill in the 3rd week?

$f(3) = \frac{102{,}000}{1+5900e^{-3}}$

Approx 346 people

c. What is the limiting size of the population that becomes ill?

102,000

Use Newton's Law of Cooling $T = C + (T_0 - C)e^{kt}$ to solve.

15. A pizza removed from the oven has a temperature of $450°F$. It is left sitting in a room with a temperature of 70°F. After 5 minutes, the temperature of the pizza is 300°F.

a. Use Newton's Law of Cooling to find a model for the temperature of the pizza T, after t minutes.

$$300 = \cancel{70} + (450 - 70)e^{k(5)}$$

$$-70 \quad -\cancel{70}$$

$$\frac{230}{380} = \frac{\cancel{380}e^{5k}}{\cancel{380}}$$

$$\frac{\ln \frac{230}{380}}{5} = k$$

$$-.1004 \approx k$$

$$\boxed{T = 70 + 380e^{-.1004t}}$$

b. What is the temperature of the pizza after 20 minutes?

$$T = 70 + 380e^{-.1004(20)}$$

$$T \approx 121°F$$

c. When will the temperature of the pizza be 140°F?

$$140 = 70 + 380e^{-.1004t}$$

$$-70 \quad -70$$

$$70 = 380e^{-.1004t}$$

$$\frac{70}{380} = e^{-.1004t}$$

$$\frac{\ln \frac{70}{380}}{-.1004} = t$$

$$16.8 \approx t$$

Approx. 17 minutes.