

Review – Polynomials (Part 2)

Name: Kathy  
 Date: \_\_\_\_\_  
 Period: \_\_\_\_\_

Percent: \_\_\_\_\_

Divide using long division. Write your answer in the form  $\frac{\text{quotient} + \text{remainder}}{\text{divisor}}$ .

$$1. (2x^3 + 9x^2 + 14x + 5) \overline{) 2x^6 + 9x^2 + 14x + 5}$$

$$\begin{array}{r} x + 1 \\ \overline{) 2x^6 + 9x^2 + 14x + 5} \\ - 2x^3 - x^2 \\ \hline 8x^2 + 14x + 5 \\ - 8x^2 - 4x \\ \hline 10x + 5 \\ - 10x - 5 \\ \hline 0 \end{array}$$

$$x^2 + 4x + 5$$

Divide using synthetic division. Write your answer in the form:  
 $(\text{divisor})(\text{quotient}) + \text{remainder}$ .

$$3. (x^3 + 27) \div (x + 3)$$

$$\begin{array}{r} 1 \ 0 \ 0 \ 27 \\ -3 \ 9 \ -27 \\ \hline 1 \ -3 \ 9 \ \cancel{0} \end{array}$$

$$(x+3)(x^2 - 3x + 9)$$

$$2. (x^5 + 1) \div (x + 1)$$

$$x + 1 \overline{) x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 1}$$

$$\begin{array}{r} x^4 - x^3 + x^2 - x + 1 \\ \overline{- x^5 - x^4} \\ x^4 + x^3 \\ \overline{- x^4 - x^3} \\ x^3 + x^2 \\ \overline{- x^3 - x^2} \\ x^2 + x \\ \overline{- x^2 - x} \\ x \end{array}$$

$$4. (x^3 - 13x - 12) \div (x - 4)$$

$$4 \overline{) 1 \ 0 \ -13 \ -12}$$

$$\begin{array}{r} 4 \ 1 \ 4 \ 12 \\ \overline{1 \ 4 \ 3 \ \cancel{0}} \end{array}$$

$$(x-4)(x^2 + 4x + 3)$$

Determine whether the given binomial is a factor of the  $p(x)$ . If so, find the remaining factors.

$$5. p(x) = x^3 - 22x^2 + 157x - 360; (x - 8)$$

$$8 \overline{) 1 \ -22 \ 157 \ -360}$$

$$\begin{array}{r} 8 \ -112 \ 360 \\ \hline 1 \ -14 \ 45 \ \cancel{0} \end{array}$$

$$(x-8)(x^2 - 14x + 45)$$

$$(x-8)(x-9)(x-5)$$

$$8, 9, 5$$

Use the rational root theorem to list all possible roots for each function. Then write the function as a product of linear factors and write all zeros.

$$6. f(x) = x^3 + 10x^2 + 32x + 32$$

$$\frac{P}{Q} = \pm 1, \pm 2, \pm 4, \pm 8 \pm 16 \pm 32$$

$$\begin{array}{r} -y \\ \hline -1 & 10 & 32 & 32 \\ & -2 & -16 & -32 \\ \hline 1 & 18 & 16 & \emptyset \end{array}$$

$$\frac{P}{Q} = \pm 1 \pm \frac{1}{2} \pm \frac{1}{4} \pm \frac{1}{8}$$

$$\begin{array}{r} 8 & 2 & -5 & 1 \\ \hline -8 & 4 & -1 \\ \hline 8 & -16 & 1 & \emptyset \end{array}$$

Solve the polynomial equation by finding all roots (rational and imaginary).

$$8. 2x^3 - 3x^2 + 8x - 12 = 0$$

$$x^2(2x - 3) + 4(2x - 3) = 0$$

$$(x^2 + 4)(2x - 3) = 0$$

$$x^2 + 4 = 0 \quad 2x - 3 = 0$$

$$x^2 = -4 \quad x = 3/2$$

$$x = \pm 2i$$

$$9. x^4 - 5x^3 + 3x^2 + x = 0$$

$$x(x^3 - 5x^2 + 3x + 1) = 0$$

$$\frac{P}{Q} = \pm 1$$

$$\begin{array}{r} 1 & -5 & -3 & 1 \\ \hline 1 & -4 & -1 & \emptyset \end{array}$$

$$x(x-1)(x^2 - 4x - 1) = 0$$

0      1      unfactorable,  
use quad. formula  
or complete the square.

$$\begin{array}{r} -y \\ \hline -1 & 10 & 32 & 32 \\ & (x+2)(x^2 + 8x + 16) \\ & (x+2)(x+4)(x+4) \\ & -2, -4 \end{array}$$

$$\begin{array}{r} (x+1)(8x^2 - 4x + 1) \\ (x+1)(8x-4)(8x-2) \\ (x+1)(2x-1)(4x-1) \\ -1 \quad 1/2, \quad 1/4 \end{array}$$

zeros:

$$-2i, 2i, 3/2$$

$$\begin{array}{r} x^2 - 4x = 1 \\ (-\frac{4}{2})^2 = (-2)^2 = 4 \\ x^2 - 4x + 4 = 5 \\ x-2 = \pm \sqrt{5} \\ x = 2 \pm \sqrt{5} \end{array}$$

zeros:  
 $0, 1, 2 + \sqrt{5}, 2 - \sqrt{5}$

Write the polynomial function, in standard form, with least degree and a leading coefficient of 1 that has the given zeros.

10.  $0, \sqrt{5}$ , and 2

$$\begin{aligned} & x(x-\sqrt{5})(x+\sqrt{5})(x-2) \\ & x(x-2)(x^2+x\sqrt{5}-x\sqrt{5}-5) \\ & x(x-2)(x^2-5) \\ & (x^2-2x)(x^2-5) \\ & x^4-5x^2-2x^3+10x \\ & (x-3)^2(x+3i)(x-3i) \\ & (x^2-(\varrho x+9))(x^2-3ix+3ix-9i^2) \\ & (x^2-(\varrho x+9))(x^2+9) \\ & x^4+9x^2-(\varrho x^3-54x+9x^2+81) \\ & = \underline{\underline{x^4-10x^3+18x^2-54x+81}} \end{aligned}$$